AP CALCULUS
LIMITS TEST ADDITIONAL PROBLEMS

Find the following limits. Do not show any improper mathematics! If a limit does not exist, perform left and right hand limits and tell whether the result is $-\infty$ or $+\infty$.

1. Evaluate $\lim\limits_{x\to 3} \dfrac{x^2+4x-21}{x-3}$
2. Evaluate $\lim\limits_{x\to 2} \dfrac{4x^2-7x-2}{x-2}$
3. Evaluate $\lim\limits_{x\to 0} \dfrac{\sqrt{4-x}-2}{x}$
4. Evaluate $\lim\limits_{x\to 9} \dfrac{x+4}{x-9}$
5. Evaluate $\lim\limits_{x\to +\infty} \dfrac{x-5x^3}{2x^3-6x+11}$
6. Evaluate $\lim\limits_{x\to 0} \dfrac{\sin 2x}{9x}$
7. Evaluate $\lim\limits_{x\to 81} \dfrac{\sqrt{x}-9}{x-81}$
8. Evaluate $\lim\limits_{x\to -\infty} \dfrac{\sqrt{7x^2+7x}}{3x+1}$
9. Find the vertical asymptotes (if any) of $f(x) = \dfrac{8}{(x-7)^3}$.
10. Find the horizontal asymptotes (if any) of $f(x) = \dfrac{7x+3}{\sqrt{5x^2-1}}$
11. Prove that the function defined by $f(x) = \dfrac{3x^2-x-2}{x-1}$ is discontinuous at $x=1$, show that the discontinuity is removable, and redefine f so that the discontinuity is removed.
12. Determine at what values of x the following function is discontinuous. Make sure you answer completely by using the test for continuity at a number.

$$f(x) = \begin{cases} x^2-4 & \text{if } x \le -2 \\ 4-x^2 & \text{if } -2 < x < 2 \\ 5-x & \text{if } x \ge 2 \end{cases}$$

13. Given the Intermediate Value Theorem holds for $f(x) = 4x^2 - 4x$ on the interval $[1, 2]$, find the value of c such that $f(c) = 3$.
14. Use the Intermediate Value Theorem to show that $f(x) = x^3 - 4x^2 + 4x + 3$ has at least one zero between $x = -1$ and $x = 2$. Do not sketch the graph. Do not find the zero(s). Use the theorem to show that a zero must exist.
15. Prove: $\lim\limits_{x\to 4} (x^2 - 3x + 4) = 8$

16. Evaluate $\lim_{x\to 0} \frac{x^6 + 5x^4}{x^4}$

17. Evaluate $\lim_{x\to 7} \frac{x^2 - 4x - 21}{x - 7}$

18. Evaluate $\lim_{x\to 0} \frac{\sqrt{9-x} - 3}{x}$

19. Evaluate $\lim_{x\to 5} \frac{x+4}{x-5}$

20. Evaluate $\lim_{x\to +\infty} \frac{3 - x + 2x^3}{5x^3 - 7x + 1}$

21. Evaluate $\lim_{x\to 0} \frac{\sin 8x}{\sin 2x}$

22. Evaluate $\lim_{x\to 49} \frac{\sqrt{x} - 7}{x - 49}$

23. Evaluate $\lim_{x\to -\infty} \frac{\sqrt{5x^2 - 2x}}{7x + 1}$

24. Find the vertical asymptotes (if any) of $f(x) = \frac{x+5}{x-2}$.

25. Find the horizontal asymptotes (if any) of $f(x) = \frac{5x - 2}{\sqrt{10x^2 + 3}}$

26. Prove that the function defined by $f(x) = \frac{x^2 - x - 20}{x - 5}$ is discontinuous at $x = 5$. Determine if the discontinuity is removable or essential. If removable, redefine f so that the discontinuity is removed.

27. Determine at what values of x the following function is discontinuous. Make sure you answer completely by using the test for continuity at a number.

$$f(x) = \begin{cases} x^2 - 9 & \text{if } x \le -3 \\ 9 - x^2 & \text{if } -3 < x < 3 \\ 5 - x & \text{if } x \ge 3 \end{cases}$$

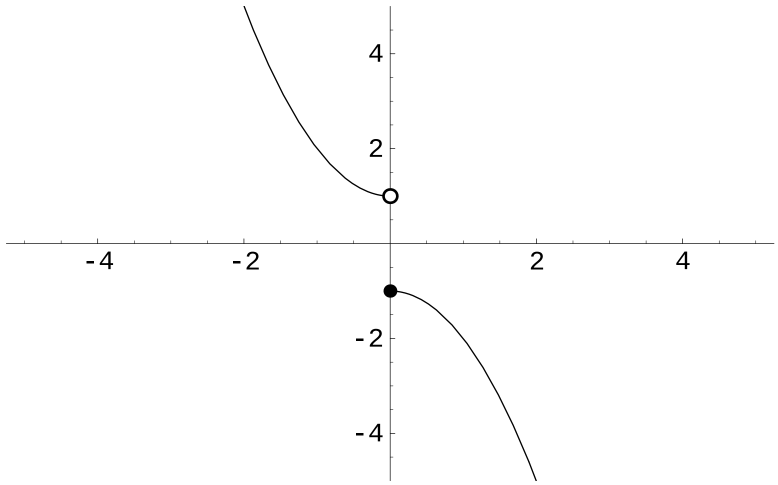
28. Given the Intermediate Value Theorem holds for $f(x) = 2x^2 + 13x$ on the interval $[0, 1]$, find the value of c such that $f(c) = 7$.

29. Use the Intermediate Value Theorem to show that $f(x) = x^3 - 3x^2 + 5x - 1$ has at least one zero between $x = 0$ and $x = 1$. Do not sketch the graph. Do not find the zero(s). Use the theorem to show that a zero must exist.

30. Prove: $\lim_{x\to 3} (x^2 - 2x + 1) = 4$

Consider the graph of the following function f. The open dot is at 1 and the closed dot is at -1.

31. Find the following. (One point each.)



(a) $\lim_{x\to 0^-} f(x)$

(b) $\lim_{x\to 0^+} f(x)$

(c) $\lim_{x\to 0} f(x))$

(d) $f(0)$

(e) State why this function is discontinuous at $x = 0$. Be specific. Use the definition of continuity.

Below are the answers…they are not complete solutions.

1. 10
2. 9
3. $-\dfrac{1}{4}$
4. $\lim\limits_{x\to 9}\dfrac{x+4}{x-9}\nexists$
   $\lim\limits_{x\to 9^+}\dfrac{x+4}{x-9}=+\infty$ and $\lim\limits_{x\to 9^-}\dfrac{x+4}{x-9}=+\infty$
5. $-\dfrac{5}{2}$
6. $\dfrac{2}{9}$
7. $\dfrac{1}{18}$
8. $-\dfrac{\sqrt{7}}{3}$
9. f has a vertical asymptote at $x=7$ because $f(7)\nexists$ and $\lim\limits_{x\to 7}f(x)=\pm\infty$.
10. Since $\lim\limits_{x\to\infty}f(x)=\dfrac{7}{\sqrt{5}}$ f has a horizontal asymptote at $y=\dfrac{7}{\sqrt{5}}$.
    Since $\lim\limits_{x\to-\infty}f(x)=-\dfrac{7}{\sqrt{5}}$ f has a horizontal asymptote at $y=-\dfrac{7}{\sqrt{5}}$.
11. Since $f(1)\nexists$ f is discontinuous at $x=1$.
    Since $\lim\limits_{x\to 1}f(x)=5$, the limit exists and the discontinuity is removable.
    $$f(x)=\begin{cases}\dfrac{3x^2-x-2}{x-1} & \text{for } x\neq 1\\ 5 & \text{for } x=1\end{cases}$$
12. Make sure you show complete continuity tests at both $x=-2$ and $x=2$.
    f is continuous at $x=-2$ because $f(-2)=\lim\limits_{x\to-2}f(x)$
    f is discontinuous at $x=2$ because $\lim\limits_{x\to 2^+}f(x)\neq\lim\limits_{x\to 2^-}f(x)\ \therefore\ \lim\limits_{x\to 2}f(x)\nexists$
13. $f(x)=3\ \longrightarrow\ x=-\dfrac{1}{2}$ or $x=\dfrac{3}{2}$ but $-\dfrac{1}{2}\notin(1,2)\ \therefore c=\dfrac{3}{2}$ only.
14. Since $f(-1)=-6<0$ and $f(2)=3>0$ by the IVT $f(x)=0$ for some $x\in(-1,2)$.
15. You should get $\delta=min\left\{1,\dfrac{\epsilon}{6}\right\}$
16. 5
17. 10
18. $-\dfrac{1}{6}$

19. $\lim_{x\to 5} \frac{x+4}{x-5} \nexists$
$\lim_{x\to 5^+} \frac{x+4}{x-5} = \infty$ and $\lim_{x\to 5^-} \frac{x+4}{x-5} = -\infty$

20. $\frac{2}{5}$

21. 4

22. $\frac{1}{14}$

23. $-\frac{\sqrt{5}}{7}$

24. f has a vertical asymptote at $x = 2$ because $f(2) \nexists$ and $\lim_{x\to 2} f(x) = \pm\,\infty$.

25. Since $\lim_{x\to\infty} f(x) = \frac{5}{\sqrt{10}}$ f has a horizontal asymptote at $y = \frac{5}{\sqrt{1}}$.
Since $\lim_{x\to -\infty} f(x) = -\frac{5}{\sqrt{10}}$ f has a horizontal asymptote at $y = -\frac{5}{\sqrt{10}}$.

26. Since $f(5) \nexists$ f is discontinuous at $x = 5$.
Since $\lim_{x\to 5} f(x) = 9$, the limit exists and the discontinuity is removable.
$$f(x) = \begin{cases} \dfrac{x^2 - x - 20}{x-5} & \text{for } \; x \neq 5 \\ 9 & \text{for } \; x = 1 \end{cases}$$

27. Make sure you show complete continuity tests at both $x = -3$ and $x = 3$.
f is continuous at $x = -3$ because $f(-3) = \lim_{x\to -3} f(x)$
f is discontinuous at $x = 3$ because $\lim_{x\to 3^+} f(x) \neq \lim_{x\to 3^-} f(x) \;\therefore\; \lim_{x\to 3} f(x) \nexists$

28. $f(x) = 7 \;\longrightarrow\; x = -7$ or $x = \frac{1}{2}$ but $-7 \notin (0,1)$ $\therefore c = \frac{1}{2}$ only.

29. Since $f(0) = -1 < 0$ and $f(1) = 2 > 0$ by the IVT $f(x) = 0$ for some $x \in (0,1)$.

30. You should get $\delta = min\left\{1, \frac{\epsilon}{5}\right\}$

31. (a) 1
(b) -1
(c) $\nexists$
(d) -1
(e) f is discontinuous at $x = 0$ because $\lim_{x\to 0} f(x) \nexists$